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A Group Single Sampling Attributes Plan to Attain a Given Strength

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Abstract: **A Single sampling attribute plan cannot be designed satisfying the given strength exactly. It is suggested that a group consisting of at most three single sampling plans be operated at random with certain proportions so that the group as a whole attains the given strength exactly. The problem of choosing at most three plans with the proportion of times they should be operated with minimum average sample size is shown to be a linear programming problem. An example with solution by simplex method is given.**

Keywords: **Sampling attribute plan, linear programming problem, given strength.**

1. INTRODUCTION

We consider an industrial situation in which a series of lots of size N items arrive at the inspection station. For acceptance sampling inspection an exact single sampling plan (SSP) (n,c) cannot be designed satisfying $Q(p_1)= 1-P(p_1) = \alpha$ and $P(p_2)= \beta$ where $p_1(p_2)$ is the satisfactory (unsatisfactory) quality level, $\alpha(\beta)$ is the producer's (consumer's) risk specified by the decision maker (DM) and $P(p)$ is the operating characteristic of the SSP (for details, see Hald [4]).For obtaining a SSP close to α and β, Chakraborty [2] modelled the problem as a fuzzy goal programming (FGP). The FGP model is shown to be equivalent to four sub-problems corresponding to the different combinations of the membership functions of Fuzzy goals. Each of these subproblems corresponds to a different type of *close to the goals* SSP according to the different combinations of the requirements of $Q(p_1)$ ($\leq, \geq$) α and $P(p_2)$ ($\leq,\geq$) β

Here we propose a system of SSP consisting of a group of atmost three plans from the set of plans corresponding to the four types of SSP's considered in Chakraborty [2] for the attainment of risks exactly at the specified levels. An individual SSP of the system is to be chosen at random for a particular lot in a specified proportion.

The idea of such group of three SSPs has been used in the MIL-STD (Hald [4]). Where a group of three SSPs is utilized for acceptance inspection though the procedure employed differs substantially from the one dealt within the present paper.

2. GROUP SINGLE SAMPLING ATTRIBUTES PLAN

A SSP from any one of the types of plans can be obtained under Poission conditions from the following general goal programming model (for notation, explanation and solution procedure, see Chakraborty [1]).

$$\text{Min } Z = P_1(\pm)\, c + P_2 w_1 d_1^{+2} + P_2 w_2 d_1^{-2} + P_3 w_3 d_2^{+2} + P_3 w_4 d_2^{-2},$$

Subject to

$$Q(p_1) + d_1^{+2} - d_1^{-2} = \alpha.$$

$$P(p_2) + d_2^{+2} - d_2^{-2} = \beta,$$

$$d_1^{+}\, d_1^{-} = 0,\ d_2^{+}\, d_2^{-} = 0.$$

Thus given (p_1,α, p_2, β) and k sets of P_j's and w_j's, one easily obtains (k>3) SSPs each of them is individually satisfactory to the DM.

Assume that the DM has chosen k SSPs (n_j,c_j), j= 1,2......k, having producer's and consumer's risks of α, β, respectively.

The problem is to determine x_j (j= 1,2...........k) the proportion of times the jth SSP should be operated at random so that the expected average sample size is minimised subject to attaining the given strengths α and β exactly. The mathematical model is the following:

This is a linear programming (LP) problem and can be easily solved by the simplex algorithm.

Note: (1) IF LP is feasible then it will provide a solution with *atmost* three x_j's >0 ⇒ Group consisting of atmost three SSPs.

(2) If LP is not feasible then this scheme cannot be utilised to obtain Group SSPs to attain the risks exactly.

Example 1. Let p_1= 0.01, α= 0.05, p_2= 0.06, β= 0.01 (Hald [4, p.50]) Let the DM choose the following five SSPs as satisfactory plans. We are to determine a group SSP.

$$\text{Minimize } x_o = n_1x_1 + n_2x_2 + \ldots.. + n_kx_k,$$

$$\begin{aligned} \text{subject to} \quad & x_1 + x_2 + \ldots.. + x_k = 1, \\ & \alpha_1x_1 + \alpha_2x_2 + \ldots.. + \alpha_k x_k = \alpha. \\ & \beta_1x_1 + \beta_2x_2 + \ldots.. + \beta_k x_k = \beta. \\ & x_1, x_2, \ldots.., x_k \geqslant 0. \end{aligned} \qquad (1)$$

S. No	nj	cj	αj	βj
1	85	2	0.055	0.116
2	90	2	0.063	0.095
3	110	3	0.026	0.105
4	120	3	0.034	0.072
5	138	3	0.052	0.035

Solving the problem by the simplex method we obtain the group SSP consisting of SSPs (85,2), (90,2) and (120,3) and these are to be operated at random with proportions 0.560, 0.147 and 0.293 respectively and will have an average sample size 96.012.

Remarks: The problem (1) can be viewed as three dimensional linear knapsack problems (see Dantzig [3]) and can also be solved by adapting and extending the centre of gravity method (see Dantzig [3, pp. 160-166]) to solve the special LP of the form (1). However the computational effort would be of the same order as that of simplex algorithm.

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